外国語科目 (数理・計算科学専攻)　　17　大修

英語

時間　午後 2 時 – 午後 3 時

注意事項

1. つぎの 3 問中 2 問を選択 し解答せよ.
2. 解答は 1 問ごとに 別々の解答用紙に記入 せよ.
3. 解答用紙ごとに必ず 問題の番号および受験番号 を記入せよ.
4. 2 問を超えて解答した場合は 採点されない 可能性がある.

問 1

次の英文は，コンピュータサイエンス分野で数学が重要であることを述べたものである．これを読んで，下の (1)(2) に答えよ．

We begin our exploration of the need for discrete mathematics in computer science with a simple problem whose solution involves its use. Vectors are supported in standard libraries of C++ and Java. From the programmer's point of view a vector looks like an extensible array. That is, while a vector is created with a given initial size, if something is added at an index beyond its extent, the vector automatically grows to be large enough to hold a value at that index.
A vector can be implemented in many ways (such as a linked list), but the most common implementation uses an array to hold the values. In such an implementation, if an element is inserted beyond its extent, the data structure creates a new array large enough to include the index, copies the elements from the old array to the new array, then adds the new element at the proper index. This vector implementation is straightforward, but how much should the array be extended each time it runs out of space?
Keeping things simple, suppose the array is being filled in increasing order, so each time it runs out of space, it needs to be extended by only one cell. There are two strategies for increasing the size of the array: always increase its size by the same fixed amount, F, and always increase its size by a fixed percentage, $P\%$. A simple analysis using discrete mathematics (really just arithmetic and geometric series) shows that in a situation in which there are many additions, the first strategy results in a situation where the average cost of each addition is $O(n)$, where n is the number of additions (that is, the total of n additions costs some constant multiplied by n^2); the average cost for each addition with the second strategy results in a constant (that is, the total of n additions costs a constant multiplied by n).
This simple but important example analyzes two different implementations of a common data structure. But we wouldn't know how to compare their quite significant differences in cost without being able to perform a mathematical analysis of the algorithms involved in the implementations.

Kim B. Bruce, Robert L. Scot Drysdale, Charles Kelemen, Allen Tucker:
Why Math? Communications of the ACM, 46 (2003) より

(1) 第二段落の下線部を和訳せよ．

(2) 二重下線部の two different implementations のうち，より優れていると著者が考えている方について，その概要を記せ．

問 2

確率論研究者の Joseph L. Doob (1910–2004) は，その著書「Stochastic Processes」(1953) について，後に次のように語っている．これを読んで下の (1)，(2) に答えよ．

I intended to minimize explicit measure theory[1] in the book because many probabilists were complaining that measure theory was killing the charm of their subject without contributing anything new. My idea was to assume as known the standard operations on expectations and conditional expectations and not even use the nasty word “measure.” This idea got me into trouble. My circumlocutions soon became so obscure that the text became unreadable and I was forced to make the measure theory explicit. I joked in my introduction that the unreadability of my final version might give readers an idea of that of the first version, but like so many of my jokes it fell flat. I was grateful that at least J. W. T. Youngs[2] noticed it, but I was less grateful that <u>it apparently mystified the Russian translators of the book</u>, who simply omitted it.

(中略)

While writing my book I had an argument with Feller[3]. He asserted that everyone said “random variable” and I asserted that everyone said “chance variable.” We obviously had to use the same name in our books, so we decided the issue by a stochastic[4] procedure. That is, we tossed for it and he won.

J. Laurie Snell: A Conversation with Joe Doob. Statist. Sci., 12 (1997) より

[1] measure theory: 測度論．解析学の一分野．現代確率論の基礎となっている．

[2] Youngs, J. W. T. (1910–1970): 数学者．専門はグラフ理論．

[3] Feller, W. (1906–1970): 数学者．専門は確率論．著書に「An Introduction to Probability Theory and its Applications Vol. I, Vol. II」(1950, 1961) がある．

[4] stochastic: 確率的な．“stochastic process” は確率過程．

(1) 下線部「it apparently mystified the Russian translators of the book」は何が原因と考えられるかを述べよ．

(2) 枠で囲まれた部分を和訳せよ (“random variable” は通常「確率変数」と訳されるが，ここでは原語のままでも構わない)．

問 3

ジャグリング（juggling：ボールやピンを規則的に投げ回す大道芸）についてのつぎの英文を読み，設問に答えよ．

Around 1985 three groups of jugglers (in Santa Cruz, California; in Pasadena, California; and in Cambridge, England) independently created the same notational system for juggling patterns. These numerical descriptions have been well publicized in the juggling world under the name siteswaps.

What are the assumptions on a notational system that led all these people to the same solution? Some of them are restrictions on how the juggler behaves:

- The juggler throws only one ball at a time, never holding more than one in each hand.
- The throws alternate hands, right left right left.
- (to avoid considering boundary conditions) The juggler has been juggling since the infinite past and will continue into the infinite future.
- The throws come one per second, with right throws at even times and left throws at odd times.

More notable, though, is the assumption about what one actually wants to record:

- The only information kept about the ball thrown at time $n \in \mathbb{Z}$ is when it is next thrown, at time $f(n) \in \mathbb{Z}$, where $\mathbb{Z}$ is the set of all integers.

In particular, this assumption loses any extra detail about throw like “under the leg”, “with a triple spin (if the objects juggled are pins rather than balls)” etc. But it suggests a very nice mathematical theory of certain functions $f : \mathbb{Z} \to \mathbb{Z}$.

(1) Siteswaps とは何かを 20 字程度で記せ．

(2) 下記に説明文のある三つのボールを用いたカスケイドとサークルジャグリングに対する関数 f を求めよ．

The three-ball cascade : Every throw is the same height, the hands alternate, every throw goes from one hand to the other, and most importantly, while each ball is in the air, exactly two other throws occur.

The three-ball circle juggling : The right hand (for most people) throws high and across and the left hand only shuttles the balls underneath, not really making a throw.

Burkard Polster 著: The Mathematics of Juggling, の Allen Knutson による書評, Notices AMS, January 2004 より．出題のため一部を編集．